Mathematics
Strategy Toolbox

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Date Modified: May 4, 2011
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**Tossed Terms**

**Purpose:** While reinforcing understanding in a fun way, the tossed terms box can prompt the recall of definitions, evoke examples, and allow students to clarify information for each other.

**Procedure:**

1. Obtain several square boxes, preferably that fit easily in one’s hand.
2. Either using the template below or on plain labels, write one vocabulary word or another lesson applicable term on each of the six “sides,” whether that is a section of the template or an individual label. Then fix the created sides around the box accordingly.
   a. The teacher may do this ahead of time or the students can in class.
3. Separate students into groups of 3-4 and have them sit or stand them in a relatively close circle.
4. One student will toss the box to another and the student now with the box will define or explain the term facing up. The rest of the circle will make sure the description is correct, adding anything else they feel is essential.
5. This process continues as long as the teacher deems necessary. Make sure the groups are monitored and walk around to help with any clarifications that may be needed.

**Differentiation:** For the words on the boxes, either the teacher may choose them and prepare the boxes ahead of time or the groups can choose and make the boxes in class. The pairings may also be planned so that particular students do not work together because of behavioral issues or so that there are a mix of advanced and lower students to implement a level of difficulty. The activity can also be turned into a challenge where one team competes against another. The first team would roll for the second and the second would have to answer. Points would then be accumulated based on right answers.

**Math Example:** For any geometry class, terms such as angle, side, circumference, radius, tangent, and line may be used. Or in a competition setting, the words add, divide, multiply, subtract, prove, simplify can be used as one team designs a problem for another team.

**Reference(s):**

Tossed Terms Box Template:
**Vocabulary Cards**

**Purpose:** While vocabulary is a key part of literacy, sometimes students need more time practicing the use of terms and more information than just the definitions. Vocabulary cards offer more than just the explanation in that they have pictures and examples that the student creates.

**Procedure:**
1. Obtain a stack of index cards, lined or unlined depending on the student. Cards should be at least 3 x 5; younger students may prefer 5 x 7.
2. Using the template provided, have the student(s) fill out the boxes accordingly. Teacher/student may decide if they want to put all 4 on one side, or split them up.
   a. Term
   b. Definition – either from the book or of their own wording
   c. Description – different than definition in that it contains characteristics or memory triggers that may help the student remember the context
   d. Visual – a drawing, picture, or example of the term
3. Use cards as review, study guides, or an assignment.

**Differentiation:** The teacher can decide to give the students all the words he/she deems necessary or students can choose which ones they want to focus on. For some students it may be best to give them the definition and then have them choose which characteristics and descriptions they think are appropriate. As students become more comfortable with using their own vocabulary, they should define the terms on their own. Students can work together or individually on them. It may help to see other classmates’ examples and drawings to inspire their own. The cards serve as great tools in chapters heavy with vocabulary or when there are similar terms.

**Math Example:** For geometry unit on triangles, for one card the student could define right triangle as “a triangle with one angle of 90 degrees.” The characteristics could include Pythagorean Theorem, two equal right triangles form a square, etc. And the visual would obviously include a drawing of such, with or without sides and angles labeled.

**Reference(s):**
## Vocabulary Cards template:

<table>
<thead>
<tr>
<th>TERM</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHARACTERISTICS</td>
<td>VISUAL</td>
</tr>
</tbody>
</table>
Word Grids / Semantic Feature Analysis

**Purpose:** An important way to develop vocabulary is by seeing the relationships between terms by their similarities and differences. By critically analyzing the definitions and characteristics, students fill in a grid for particular terms while noting the associations among the objects.

**Procedure:**
1. Create a grid where topics can be listed in the first column and characteristics along the first row (a template, along with an example, is provided).
2. Then using the topic of the lesson suggest related terms that should be categorized.
3. As a class, identify the terms by their characteristics while explaining why each category is necessary. Put an X or check mark in each category it fulfills.
4. The students can then continue on their own by working through each word one by one, while jotting down relationships between the terms.
5. A master grid can be created for the whole class to be filled in as the unit goes on, students will be able to make the connections between what they have been completing and the material that may be tested upon.

**Differentiation:** To develop vocabulary comprehension, word grids identify particular features that may relate between a variety of terms or topics. Depending on the level of difficulty desired, the teacher may provide the categories to be sorted into or the students can come up with them on their own. After modeling an example or two, students will begin to work on their own and compare it to a master chart. This then becomes an easy study guide.

**Math Example:** See the Word Grid provided for an example of how quadrilaterals may be semantically analyzed.

**Reference(s):**
Word Grid Template and Example:

<table>
<thead>
<tr>
<th>Quadrilateral Classification</th>
<th>(at least) 1 pair of parallel sides</th>
<th>2 pairs of congruent sides</th>
<th>4 right angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trapezoid</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parallelogram</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rhombus</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Rectangle</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Square</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Kite</td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>
Word Sorts

Purpose: A crucial part of developing vocabulary is being able to see the relationships between words. Word Sorts is an activity allowing content to be connected and expanded upon while manipulating a set of words into a series of categories.

Procedure:
1. Choose around 10-20 terms to be sorted into categories, in one of the ways listed:
   a. Closed: where the teacher determines which categories the words should be sorted in, places an emphasis on the students’ classification skills
   b. Open: students choose their own categories and may vary group to group, places an emphasis on understanding relationships between terms
2. Model a brief example in which you move words from category to category, explaining your thinking and understanding of the word.
3. Allow students to work individually or in groups of 2-3, depending on which they need, and place a list of the words on the board in no particular order.
4. Have the students copy down the words onto index cards or pieces of paper cut up and allow them to separate the words into the categories. As they do, walk around and observe while asking students why they chose their placements.
   a. For further practice with words, have the students try to define the word on the back of the paper.
5. It is also a good idea to have students write down their sorts with a brief explanation of their thinking that went along with the sorting.

Differentiation: This activity is great to develop either autonomy or promote interaction. It expands their critical thinking, especially if they have to come up with the categories, but there is no “wrong” answer. As long as the student can justify their decision of where to put the word, it strengthens their ability to understand concepts. Word Sorts may also help review previous material and/or connect it to a newer lesson.

Math Example: If the list of words provided was: triangle, line, rectangle, hexagon, star, circle, circumference, semicircle, diamond, pentagon, angle, and radius, potential categories could be “circle related” and “polygons” or “things to do measurement.” In this case, drawing or defining the terms on the back may be a helpful tool in reviewing the material.

Reference(s):
Directed Notetaking Activity

**Purpose:** Notes are a key component of a student’s information artifacts. Using Directed Notetaking, students learn how to organize material from the class and study more efficiently from their notes.

**Procedure:**
1. While there are several variations of notes, it is helpful for students to know how to be proficient with their notes, especially in different subjects and if they are just entering high school. Provided is a list of guidelines that may help students.
2. The first time, as you give the lesson or lecture, use an overhead to project the notes you would be taking as if you were listening to the lecture. To save time, have these notes prepared ahead of time and present them little by little.
3. Add questions or other thoughts as you go on with the lesson, along with input from students.
4. As you teach, pause often to allow students to write down notes.
5. Afterwards, ask students questions based on the notes to have them get into practice of reviewing the material.

**Differentiation:** The modeling aspect helps students see a professional display of notes. Through scaffolding and practice, eventually students will be able to take notes on their own that are organized and can easily serve as study guides. It may even help them in other classes where they struggle to take proper notes and identify main ideas.

**Math Example:** Provided is an example of geometry notes using different colors, indentations, and set ups that can be modeled so students follow a similar trend.

**Reference(s):**
**Example of Notetaking Guidelines:**

<table>
<thead>
<tr>
<th>As you enter class/get ready to take notes:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Have your notebook organized, with dividers and full supply of paper</td>
</tr>
<tr>
<td>• Have your paper and writing implement(s) ready</td>
</tr>
<tr>
<td>• Look over the notes from the previous lesson</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>During the lesson/as you take notes:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Stay within the margins</td>
</tr>
<tr>
<td>• Skip lines to indicate new ideas</td>
</tr>
<tr>
<td>• Bullet, indent, number, and letter information as appropriate</td>
</tr>
<tr>
<td>• Listen carefully to the information and write down important ideas and their details</td>
</tr>
<tr>
<td>• Only use abbreviations you know you will remember</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>After taking notes:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Review the notes, identifying important ideas</td>
</tr>
<tr>
<td>• Highlight or underline vocabulary terms</td>
</tr>
</tbody>
</table>
Chapter Two, Section One (If-Then Statements; Converes)
Your friend says, "If it rains after school, then I will give you a ride home."

These are examples of if-then statements, which are also called conditional statements or simply conditionals.

To represent an if-then statement symbolically, let $p$ represent the hypothesis, and let $q$ represent the conclusion. Then we have the basic form of an if-then statement shown below:

$$p \implies q$$

$p$: hypothesis

$q$: conclusion

The converse of a conditional is formed by interchanging the hypothesis and the conclusion.

Statement: If $p$, then $q$

Converse: If $q$, then $p$

A statement and its converse say different things. In fact, some true statements have false converses. An if-then statement is false if an example can be found for which the hypothesis is true and the conclusion is false. Such an example is called a counterexample. It takes only one counterexample to disprove a statement.

Conditional statements are not always written with the "if" clause first.

<table>
<thead>
<tr>
<th>General Form</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $p$, then $q$.</td>
<td>If $x^2 = 25$, then $x &lt; 10$.</td>
</tr>
<tr>
<td>$p$ implies $q$.</td>
<td>$x^2 = 25$ implies $x &lt; 10$.</td>
</tr>
<tr>
<td>$p$ only if $q$.</td>
<td>$x^2 = 25$ only if $x &lt; 10$.</td>
</tr>
<tr>
<td>$q$ if $p$.</td>
<td>$x &lt; 10$ if $x^2 = 25$.</td>
</tr>
</tbody>
</table>
Split-Page Notetaking

Purpose: An example of Directed Notetaking is the Split-Page method where students organize notes by separating big ideas from supporting details, while promoting active listening.

Procedure:
1. Have students write down the lesson topic at the top of their page along with the date, then have them draw a straight line down the page, about an inch and a half or two inches from the left margin.
2. In the left column they will write down the BIG IDEAS including: terms, dates, names, etc., but this column should have little information and more along the lines of topics.
3. On the right, they should add corresponding facts, processes, and details corresponding to the big ideas listed. Paraphrasing will be key in keeping notes efficient.
4. Discuss the benefits of note-taking this way, especially how easy it is to study by folding the paper down the line drawn.

Differentiation: Similarly to the Directed Notetaking Activity, modeling and scaffolding will be necessary to help students get in the habit of this method. It also allows students to become familiar with what is a big idea in comparison to a supporting detail. After notes are taken, students will have a simple pre-made study guide where they should know the supporting details for the big ideas. It also creates a sense of ownership over their notes, thus increasing the chance they will remember what they wrote down.

Math Example: When it comes to factoring there are several methods one could use. Split-page notes can help separate the different methods of substitution, completing the square, etc. It may even be helpful to have a sub-column to separate the method, the steps required, and an example broken down by each step.

Reference(s):
Think-Alouds

**Purpose:** Think-Alouds get students to think more critically about what they are reading or the process they are carrying out. It forces students to engage in the material while considering their own thought processes at the time.

**Procedure:**
1. Provide the class with an excerpt or word problem.
2. As you read or solve the problem describe out loud your thought processes as you modeling the example.
3. Have students to continue on with the reading or excerpt with a partner while jotting down notes.
4. Come together as a class to discuss strategies, any thoughts that came to mind as they did the assignment, realizations, and possible other ways/methods.

**Differentiation:** This method begins with modeling so students get the idea of how to think aloud themselves. By working with a partner, not only can they demonstrate their own understanding but learn from those of their peers as well. Not each student will be as vocal, or content literate as their pair, so it helps to start small and learn the habit before being sent off to do it on their own and get the assignment correct.

**Math Example:** When learning how to FOIL, the mnemonic device has a particular process. By talking yourself through the steps of which numbers to multiply first, then the outside, followed by the inside, and ending with the last terms you are able to see your work. If you just jumped on in, you may skip a step or make a sign error easily.

**Reference(s):**
Adjunct Displays

Purpose: Adjunct Displays provide students with two ways to remember material: verbally through the text and spatially in relation to the other information. Adjunct Displays are also known as Graphic Organizers: concept maps, cycles, decision trees, flow charts, matrices, shape maps, Venn diagrams, etc.

Procedure:
1. Provide students with a blank or partially completed graphic organizer
2. Discuss the main ideas or themes from the topic, explaining why such organizer will be helpful to students
3. Explain the purpose of the activity, whether that is for developing a summary, writing an essay, giving a presentation, for their notes, etc.
4. As students read or complete the activity, assist them with completing their displays, each one will be different
5. Have students pair up or in a group to collaborate, help each other, and review the information verbally and by writing down notes

Differentiation: By providing an adjunct display, students will be able to see the information they are learning structured in a way similar to that of the organizer. As they continue through the activity they fill in the organizer with both big ideas and supporting details. Each one will be different allowing for a sense of autonomy and individual understanding and then working with others will help them fill in missed details and see other perspectives.

Math Example: See the example provided of how a Venn Diagram may be used in Geometry to relate quadrilaterals (for further practice have the students add definitions and drawings of the shapes).

Reference(s):
Adjunct Display: Venn Diagram Example

Venn Diagram

- Parallelograms
  - Rectangles
  - Squares
  - Rhombuses
- Quadrilaterals
  - Trapezoids
KWL Chart

**Purpose:** KWL charts help focus the lesson on the interests and prior knowledge of students within the topic by asking the questions “what do you Know?” “what do you Want to know?” and “what have you Learned?”

**Procedure:**
1. Create a chart of 3 columns for the K,W, and L respectively.
   a. Other columns for H (how do we know), Q (questions), and A (how does this affect us) may be added if desired.
2. Ask students, either individually or with a partner, to write down what information they know or think they know about the topic in the K column and anything they want to know or think they need clarification on in the W column.
3. As a whole class, record the groups’ answers on the board/overhead, taking special note of what the students want to know.
4. Proceed with the lesson, slightly tailoring it to what the students have told you.
5. Afterwards, have the students return to their charts filling in the final column(s) based on their notes and the lesson.
   a. Provided is a rubric created by Fisher et al. if the KWL is to be graded.

**Differentiation:** To get students engaged, it is sometimes fun to pass out different colors of paper and markers to have the students make their own charts. Some students know that they will need less space for what they want to know and more for what they know or have learned. It is also often helpful to have the students fill out the chart the day before a lesson so that you can decided where to proceed after seeing their input, this gives students an opportunity to influence their own learning and proceed with knowledge, rather to have repeating material. At the end, the charts can be collected as an assessment assignment and to address any areas that need further clarification.

**Math Example:** For a lesson in graphing nonlinear equations, it can be a helpful review to go over everything students know about nonlinear equations. They can then jot down what they think they know about graphing them and how they may want to graph the possible shapes. The lecture may then consist of explaining circles, ellipses, and parabolas. Afterwards the L column may be like a mini assessment to see if students can graph equations given.

**Reference(s):**
KWL Grading Rubric (Guide):

1. No Evidence
   a. No attempted made to fill in chart
   b. Does not turn in project for grading

2. Progressing
   a. Chart is partially completed
   b. Answers less than half of the W questions in the L seconds and includes information in the written assignment

3. Meets Standards
   a. Completes the KWL chart with 3-4 entries in each column
   b. Demonstrates adequate writing skills with proper grammar and punctuation in the writing assignment

4. Exceeds Standards
   a. Chart has 5-6 entries in each column
   b. Above average writing skills, but no new information in the writing section

5. Exemplary
   a. Completed chart with 7+ entries in each column
   b. Superb writing skills with new information in the writing piece
Mnemonic Devices

Purpose: Mnemonics are created to help students learn and remember individual pieces of information, sets of information, procures, or general ideas. They give simple reminders to aspects that require more elaboration or processing.

Procedure:
1. Identify what information or area needs to be remembered distinctly.
2. The key part is to make each letter of the individual content aspect match up with the letter of a common word.
   a. This can be done to either be one word “H.O.M.E.S” (like for the Great Lakes) or a sentence of easily remembered words “My Very Excited Mother Just Served Us Nine Pizzas” (for the planets).
3. Many mnemonics can be found online.

Differentiation: As the student becomes familiar with the Mnemonic and the information associated with it, they can create their own. These are very helpful for students who have trouble with memorization or just need a more entertaining way to remember material, especially processes.

Math Example: One of the most common math processes uses a Mnemonic device, although many people still get math problems wrong when they forget to use such. PEMDAS, also known as “Please Excuse My Dear Aunt Sally,” is the order in which to solve a problem using the order of operations. The proper order is: parentheses, exponents, multiplication/division, and addition/subtraction. We use the slashes because those particular operations can be done either multiplication then division, or division then multiplication, whichever is easiest. Mnemonics are helpful because students can jot down their little phrase in the corner of a quiz or test as it helps them remember the information.

Reference(s):
Pattern Guide

**Purpose:** Pattern guides are used to show the relationships between concepts, while allowing students to develop an appreciation for hands-on activities. Information that has cause and effect, compare and contrast, temporal sequence, problem/solution, or descriptive structures are best for using such guides.

**Procedure:**
1. Choose a passage or section that would require a pattern guide (based on the information above), particularly concepts with structures or prominent buzz words.
2. Develop a pattern guide of interlocking pieces similar to the example provided.
3. Model and scaffold an example first of how a relationship or pattern may be found from the text.
4. As students read, either together or individually, have them assemble their own guide by looking for examples from the text that seem to correspond in any pattern.
5. After going over some of the pieces as a group students may:
   a. Put the guide in their notes
   b. Redistribute the pieces among a group and have them reassemble the order
   c. Give an oral presentation or develop a written piece on the order / relationship they found

**Differentiation:** Students are encouraged to really focus on the material and develop a relationship from what they have read if they cannot create one from the passage it is a sign that they should go back and look more closely at different aspects. Struggling students can work with a partner in finding the relationships, but then describing it in their own words. When the class comes together they will see a variety of flowing information.

**Math Example:** When factoring powers of 3 or more, there is a routine to follow. First identify the power and realizing if all terms are cubed, difference of squares, or if it needs to be mirrored. Eventually all of these processes conclude the same way, but a pattern guide can help show how the arrows direct each different power to one end step.

**Reference(s):**
Pattern Guide Template:
Popcorn Review

Purpose: The routine of a Popcorn Review involves students popping up out of their seats to state a fact, detail, or read a passage. The students take responsibility for their learning and that of their peers as they prompt each other and participate for the good of a whole.

Procedure:
1. Arrange the room in a way in which students can all see each other easily, or in small groups.
2. Present the group with a topic, problem, or passage and have one student state a relating detail, do a step of the procedure, or start reading respectively.
3. After the first student says their piece, another randomly pops up and continues on with another factor, a subsequent detail to the process, or continues the reading.
4. Continue with varying topics and examples, allowing all students to get a chance.

Differentiation: Not only is this a fun review process, but it allows students to share information they know and are confident with. If you put constraints on how many times a student can pop up, students who are quieter are then forced, in a good way, to participate so that the whole group can learn from each other. Hearing ideas and from others may spark a new thought process and motivate participation.

Math Example: When going over angles and segments in circles, popcorn review can be a less pressured way to complete a proof. If the teacher started with “if two tangent lines are drawn on a circle and they cross, the lengths of the two tangents will be the same,” students could pop up with theorems and postulates they know to make that statement true, eventually going further to prove other aspects of the same circle. It can even help to draw the circle, along with the proven lines, on the board.

Reference(s):
**Professor Know-It-All**

**Purpose:** In this activity, students act in the role of “experts” on a particular topic demonstrating their vast knowledge. Students have to prepare thoroughly as their comprehension will be questioned and challenged.

**Procedure:**
1. After a content unit or topic has been comprehensively covered, select a group of 3-4 students to be “the Professor” and allow them time to review the material.
2. The rest of the group will come up with questions about the content they want clarified, may be on a future assessment, or just asking on the material.
   a. A first few questions and answers may be modeled by the teacher to show the need for both factual questions and those requiring deep thinking.
3. The group is called to the front of the room and must stand shoulder to shoulder as the first question is asked.
4. For the answer, each person of the Professor can only say one word as they continue down the line until an answer, correct or not, is given in proper sentence form.
   a. Students may decide that they want to huddle first to discuss an answer or go right into it.
5. Continue as deemed appropriate, allowing as many students as possible to participate as the Professor.

**Differentiation:** This method increases student responsibility for their learning. As they prepare for the role, they are pressured to really know the material and be an expert. By working in a group, students support each other and prompt answers, so even if one person forgets a word, another can easily save the game. The teacher can model questions so students know what to expect and the range of difficulty.

**Math Example:** This can be a fun method describing shapes and their characteristics. “A-semicircle-only-has-one-hundred-and-eighty-degrees-in-comparison-to-a-full-circle-with-three-hundred-and-sixty.”

**Reference(s):**
Student Questions for Purposeful Learning (SQPL)

Purpose: Each teacher hopes to teach with a purpose so that there is a reason for students to learn the material. SQPL promotes student engagement and involvement as it creates meaningful learning, eliminates fake participation, and requires genuine attention.

Procedure:
1. Come up with a statement relating to the lesson that would cause students to wonder about, want to know more, and question.
   a. It does not necessarily have to be true or long, but thought provoking.
2. Present the statement either orally, written on the board, or passed out by handout.
3. Have students pair up, talk about the statement, and jot down their curiosities and questions related to it.
4. Ask each group to write one of their questions or thoughts on the board. Have students write down what is on the board and come back together for a full classroom discussion.
5. After talking about what they thought, go into the lesson, mentioning they should pay attention for the answers to their questions. Stop periodically so that they may add thoughts to their notes relating to the questions.

Differentiation: This strategy prompts students to focus in on their learning and explore further with their invested attention. By working in pairs students can share thoughts at a lower level before going in front of a whole group. As they copy down other students’ questions it may trigger some more of their own. Then as the lesson continues they have a desire to answer the questions and expand with their reactions. It allows each student to add their thoughts at their own pace throughout the lesson, whenever the realizations may occur.

Math Example: An example statement may be “with just a ruler I can tell you exactly how many steps it will take me to get from here to China.” Students will then question miles, measurement, shoe size, stride, length of time required, etc. It can also lead into an experimental type of activity along with the lesson.

Reference(s):